Final Exam

Philosophical Logic 2025/2026

Exercise 1 [30 points]

Show that

1. **Fuzzy Logic $Ł_{\aleph_1}$:** $p \to (p \to q) \nvDash_{\text{deg}} p \to q$
   (truth values $[0, 1]$, Łukasiewicz semantics, degree consequence)

2. **Truthmaker semantics:** $p \wedge p \vDash_{\text{TM}} p$

3. **Supervaluations** (global consequence relation):
   the following meta-inference (conditional proof) fails:
   if $\varphi \vDash_g \chi$ and $\psi \vDash_g \chi$, then $\varphi \vee \psi \vDash_g \chi$.
   (i.e., find formulas φ, ψ, χ s.t. $\varphi \vDash_g \chi$ and $\psi \vDash_g \chi$ but $\varphi \vee \psi \nvDash_g \chi$)
   Hint: you may use without proof that for any φ, $\varphi \vDash_g \Delta\varphi$.

4. **Non-monotonic logic:** the following rule is derivable in **P**:
   If $\varphi \mathrel{|\!\sim} \psi \supset \chi$ and $\varphi \mathrel{|\!\sim} \psi$, then $\varphi \mathrel{|\!\sim} \chi$,
   where $\supset$ is the material conditional $\varphi \supset \psi \equiv \neg\varphi \vee \psi$
   (i.e., you need to provide a proof-theoretic derivation of $\varphi \mathrel{|\!\sim} \chi$ in **P** taking $\varphi \mathrel{|\!\sim} \psi \supset \chi$ and $\varphi \mathrel{|\!\sim} \psi$ as additional axioms; you are not allowed to use completeness of preferential consequence)

5. **Probability:** $p \to q, r \to l \vDash_P (p \vee r) \to (q \vee l)$, where '$\to$' is the indicative conditional, defined using conditional probability $P(\varphi \to \psi) = P(\psi|\varphi) = \frac{P(\psi \wedge \varphi)}{P(\varphi)}$.

   Tip: The last two facts of Exercise 1 require more effort. You might consider working on the subsequent exercises and returning to Exercise 1 afterwards.

Exercise 2 [20 points]

Consider the Łukasiewicz Ł3 three-valued logic.

1. Show that the binary connective $*$ defined by the truth table below is not expressible in Ł3. In particular, show that for any formula φ whose only sentence letters are p and q and has no other connective besides $\neg$, $\vee$, $\wedge$ and $\to$, there is a valuation v s.t. $v(\varphi) \neq v(p * q)$.

2. Show that the binary connective $\vee_w$ (the Weak Kleene disjunction) defined by the truth table below is expressible in Ł3. In particular, find a formula φ which contains only the sentence letters p, q and the connectives $\neg$, $\vee$, $\wedge$, $\to$, s.t. $\varphi \equiv p \vee_w q$. Motivate your answer.

$*$	1	i	0
1	i	i	0
i	i	i	0
0	0	0	0

$\vee_w$	1	i	0
1	1	i	1
i	i	i	i
0	1	i	0

Exercise 3 [25 points]

This exercise concerns similarity analysis of counterfactuals.

Frames: $F = \langle W, \{\prec_w\}_{w \in W} \rangle$ with each $\prec_w$ a strict partial order on $W_w \subseteq W$.

Connectedness: for all w, for all $u, v \in W_w$, either $u = v$, or $u \prec_w v$, or $v \prec_w u$

CEM: $(\varphi \rightsquigarrow \psi) \vee (\varphi \rightsquigarrow \neg\psi)$.

1. Show that, under the Limit Assumption, Connectedness holds *iff* CEM is valid on F:
   (a) Connectedness $\Rightarrow$ CEM
   (b) Connectedness $\Leftarrow$ CEM
2. Drop the Limit Assumption. Show that the $\Rightarrow$ direction fails, while the $\Leftarrow$ direction still holds:
   (a) Connectedness $\not\Rightarrow$ CEM
   (b) Connectedness $\Leftarrow$ CEM

Exercise 4 [25 points]

According to *truthmaker maximalism*, every truth is made true by some portion of reality.

Which problems do statements like "Unicorns do not exist" pose for truthmaker maximalism? What arguments can a truthmaker maximalist use to accommodate such statements while still maintaining a maximalist position? Do you find such possible arguments convincing? Motivate your answer.